CHE260: Thermodynamics Midterm

QiLin Xue

Fall 2021

1 Basic Thermodynamics

- The ideal gas law
$$PV = mRT \tag{1}$$
- The first law of thermodynamics applies to **closed systems only**:
$$U = Q + W \tag{2}$$
- For an ideal gas,
$$c_p = c_v + R \tag{3}$$
$$\Delta u = c_v \Delta T \tag{4}$$
$$\Delta h = c_p \Delta T \tag{5}$$
- For a liquid or solid
$$\Delta h = c\Delta T + v\Delta P \tag{6}$$
- For a control volume
$$\dot{m} = \rho A V \tag{7}$$
$$\dot{Q} + \dot{W} = \dot{m} \tag{8}$$
and we get the energy rate balance
$$\left(\Delta h + \frac{V_2^2 - V_1^2}{2} + g\Delta z\right) \tag{9}$$
- At steady state, we have
$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \tag{10}$$
- The work done in a polytropic process $(PV)^n =$ constant is
$$W_{12} = \begin{cases} P_1 V_1 \ln \dfrac{V_1}{V_2} & n = 1 \\ \\ \dfrac{P_2V_2 - P_1V_1}{n-1} & \neq 1 \end{cases} \tag{11}$$

2 Entropy

- Entropy can be defined as
$$\mathrm{d}S = \frac{\mathrm{d}Q}{T} + \mathrm{d}S_{\text{gen}} \tag{12}$$
- Entropy can also be defined statistically to be
$$S = k_B \ln \Omega \tag{13}$$
where k_B is Boltzmann's constant and Ω is the number of microstates.

- The change in entropy of an ideal gas with constant specific is given by (all three are equivalent):

$$\Delta s = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_1}{v_2} \tag{14}$$
$$\Delta s = c_v \ln \frac{P_2}{P_1} + c_p \ln \frac{v_2}{v_1} \tag{15}$$
$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \tag{16}$$
$$\Delta s = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \tag{17}$$

  where s_2°, s_1° are standard specific enthalpy values that can be searched up using a table.

- For an adiabatic and isentropic process, the quantity Pv^γ is constant.
- We can perform an entropy rate balance. We have,

$$\frac{dS}{dt} = \dot{S}_{\text{in}} - \dot{S}_{\text{out}} = \dot{S}_{\text{gen}} \tag{18}$$
$$\frac{dS}{dt} = \sum \frac{\dot{Q}}{T} + \sum \dot{m}_{\text{in}} s_{\text{in}} - \sum \dot{m}_{\text{out}} s_{\text{out}} + \dot{S}_{\text{gen}}. \tag{19}$$

  At steady state, $\frac{dS}{dt} = 0$.

- We can write Gibb's Equation in the following forms

$$T\,\mathrm{d}s = \mathrm{d}u + P\,\mathrm{d}v \tag{20}$$
$$T\,\mathrm{d}s = \mathrm{d}h - v\,\mathrm{d}P \tag{21}$$

- Turbine efficiency:

$$\eta_{\text{turbine}} = \frac{h_2 - h_1}{h_{2s} - h_1} \tag{22}$$

  and compressor efficiency:

$$\eta_{\text{compressor}} = \frac{h_{2s} - h_1}{h_s - h_1} \tag{23}$$

  Remember that turbines create energy so they want to maximize work. Compressors focus on compression using the least amount of work.

- For an isentropic and incompressible process (isothermal), we get Bernoulli's equation:

$$\frac{P}{\rho} + \frac{v^2}{2} + gz = \text{constant} \tag{24}$$

3 Phase Change

- The Clapeyron Equation gives

$$\frac{dP}{dT} = \frac{h_{fg}}{T(v_g - v_f)} \tag{25}$$

  where $h_{fg} \equiv h_g - h_f$. Assuming an ideal gas, we get the Clausius-Clapeyron equation:

$$\frac{dP}{dT} = \frac{h_{fg} P}{RT^2} \tag{26}$$

- For phase equilibrium, Gibb's equation becomes

$$s_g - s_f = \frac{h_g - h_f}{T} \tag{27}$$

  since pressure and temperature are constant for a system in equilibrium.

- The saturation pressure can be written as

$$P_{\mathsf{sat}} = C \exp\left(-\frac{h_{fg}}{RT_{\mathsf{sat}}}\right) \tag{28}$$

- The quality of a mixture is defined aS

$$x = \frac{\text{mass of vapour}}{\text{mass of mixture}} = \frac{m_g}{m} \tag{29}$$

- Suppose ξ is an intensive quantity. The intensive quantity ξ of a mixture is

$$\xi = \xi_f + x(\xi_g - \xi_f) \tag{30}$$

4 Heat Engines

- The thermal efficiency of a heat engine is

$$\eta_{\mathsf{th}} = \frac{W_{\mathsf{net}}}{Q_{\mathsf{in}}} \tag{31}$$

- The thermal efficiency of a carnot engine is

$$\eta_{\mathsf{th,carnot}} = 1 - \frac{Q_C}{Q_H} - 1 - \frac{T_C}{T_H} \tag{32}$$

- The coefficient of performance of a refrigerator is

$$COP_R = \frac{Q_C}{W_{\mathsf{net}}} = \frac{1}{T_H/T_C - 1} \tag{33}$$

  and the coefficient of performance of a heat pump is

$$COP_{HP} = \frac{Q_H}{W_{\mathsf{net}}} = \frac{1}{1 - T_C/T_H} \tag{34}$$

- For cyclic cycles, $\Delta U = 0$, so the work that an engine does is $W = Q_h - Q_c$.