CIV102: Finals Review

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Fall 2020

1 Trust

- To be a safe bridge, we want to prevent tension/compression failure by demanding the cross sectional area be:

$$A \geq \frac{2F}{\sigma_y} \tag{1}$$

where σ_y is the yield strength and for steel, it is the same for both compression and tension.

- To prevent compression members from buckling, the moment of inertia needs to be:

$$I \geq \frac{3FL^2}{\pi^2 E} \tag{2}$$

- And we also demand the radius of gyration r to be:

$$r \geq \frac{L}{200} \tag{3}$$

- To use the method of virtual work, replace all applied forces with a single force F^* at the location of interest and solve for the forces in all the members P^*. Then the displacement is:

$$\Delta = \frac{1}{F^*} \sum \frac{PP^*L}{AE} \tag{4}$$

- For a point load at midspan, the frequency of oscillations is:

$$f_n = \frac{15.76}{\sqrt{\Delta}} \tag{5}$$

For a uniformly distributed load, we have:

$$f_n = \frac{17.76}{\sqrt{\Delta}} \tag{6}$$

2 Beem

- Navier's equation is:

$$\sigma = \frac{My}{I} \tag{7}$$

- The curvature is:

$$\phi = \frac{M}{EI} \tag{8}$$

- The change in slope between two points is given by the **first moment area theorem**:

$$\Delta_{AB} = \theta_B - \theta_A = \int_A^B \phi(x)\,\mathrm{d}x \tag{9}$$

- The **second moment area theorem** gives the deviation of point D from the tangent drawn at point T as:

$$\delta_{DT} = \int_D^T x\phi(x)\,\mathrm{d}x = \bar{x}_{DT}\int_D^T \phi(x)\,\mathrm{d}x = \sum \bar{x}\int \phi(x)\,\mathrm{d}x \tag{10}$$

- There are three scenarios when finding the deflection:
  - Known horizontal tangent due to support: Find the deflection of the point of interest from the support condition.
  - Known horizontal tangent due to symmetry: Find the deflection of the support from the point of interest.
  - No known horizontal tangents: Find the deflection of support C from the tangent drawn from the other support A. Find the deflection of the point of interest from support A. Use similar triangles to relate them together. To calculate the deflection of the support from A, we have:

$$\theta_A = \frac{\delta_{CA}}{L} \tag{11}$$

- Shear stresses are given by **Jourawski's equation**:

$$\tau = \frac{VQ}{Ib} \tag{12}$$

where:

$$Q = \int_{y_{\text{bot}}}^{y} y\,\mathrm{d}A = \int_{y}^{y_{\text{top}}} y\,\mathrm{d}A = \sum Ad \tag{13}$$

- Plate buckling equations are given below:

Table 30.2 – Summary of plate buckling failure modes

No.	Failure Mode	Failure Condition	Relevant Design Equation
5	Buckling of the compressive flange between the webs	$\sigma = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	$\sigma = \frac{My}{I}$
6	Buckling of the tips of the compressive flange	$\sigma = \frac{0.425\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	
7	Buckling of the webs due to the flexural stresses	$\sigma = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$	
8	Shear buckling of the webs	$\tau = \frac{5\pi^2 E}{12(1-\mu^2)}\left(\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right)$	$\tau = \frac{VQ}{Ib}$

3 Kuancrete

3.1 Properties

- The **concrete compressive strength** f_c' and the **concrete tensile strength** f_t' is related by the relationship:

$$f_t' = 0.33\sqrt{f_c'} \tag{14}$$

- The Young's modulus of the concrete E_c can be correlated to the compressive strength of concrete via:

$$E_c = 4730\sqrt{f_c'} \tag{15}$$

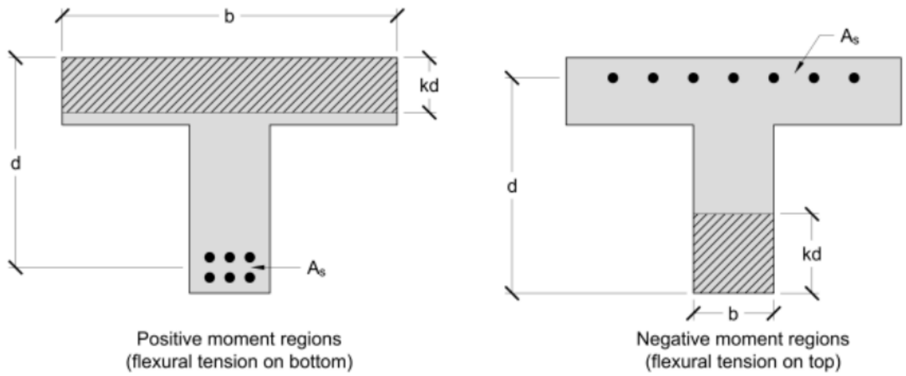
- For *reinforcing steel*, the Young's modulus is $E_s = 200,000\text{MPa}$ and the yield strength is $f_y = 400\text{MPa}$.
- The **modular ratio** n is given as:

$$n = \frac{E_s}{E_c} \tag{16}$$

- The **quantity of longitudinal reinforcement** ρ is given as:

$$\rho = \frac{A_s}{bd} \tag{17}$$

where A_s is the area of the steel reinforcements, b is the width of the cross sectional region of interest and d is the distance from the top/bottom edge of the region of interest to the opposing reinforcements. Refer to the below diagram:



- The value of k (scaling factor such that kd is the distance from the extreme compression fibre to the neutral axis) is given as:

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{18}$$

- The value of j (scaling factor such that the **flexural lever** jd is the vertical distance between the compressive and tensile forces) is given as:

$$j = 1 - \frac{1}{3}k \tag{19}$$

- If the values of k and j are unknown, let $k = \frac{3}{8}$ and $j = \frac{7}{8}$.

3.2 Flexural Stress Analysis

- The stress in the reinforcement f_s is given by:

$$f_s = \frac{M}{A_s jd} \tag{20}$$

where M is the bending moment carried by the member.

- The stress in the concrete f_c is given by:

$$f_c = \frac{k}{1-k}\frac{M}{nA_s jd} \tag{21}$$

- The maximum moment which can be carried by the member if it fails by yielding, M_{yield} is given by:

$$M_{\text{yield}} = A_s f_y jd \tag{22}$$

- Perform the following tests to see if the concrete is safe:
  - $A_s \geq \frac{M}{0.6 f_y jd}$
  - $f_s \leq 0.6 f_y$
  - $f_c \leq 0.5 f'_c$

3.3 Shear Stress Analysis

- The maximum shear stress v we need to design for in a cracked concrete member occurs in its web, and is given by:

$$v = \frac{V}{b_w jd} \tag{23}$$

where b_w is the effective web width.

- The shear stress v_{max} that causes buckling to take place from the diagonal compression is given by:

$$v_{\text{max}} = 0.25 f_c' \tag{24}$$

- **With no shear reinforcement:** The shear strength of the concrete without shear reinforcement is given by:

$$V_c = \frac{230\sqrt{f_c'}}{1000 + 0.9d} b_w jd \tag{25}$$

- **With shear reinforcement:** These are created with stirrups, and the shear strength in the concrete is given by:

$$V_c = 0.18\sqrt{f_c'} b_w jd \tag{26}$$

This equation is valid if:

$$\frac{A_v f_y}{b_w s} \geq 0.06\sqrt{f_c'} \tag{27}$$

where A_v is the effective area of the stirrups. See the below diagram for different configurations:

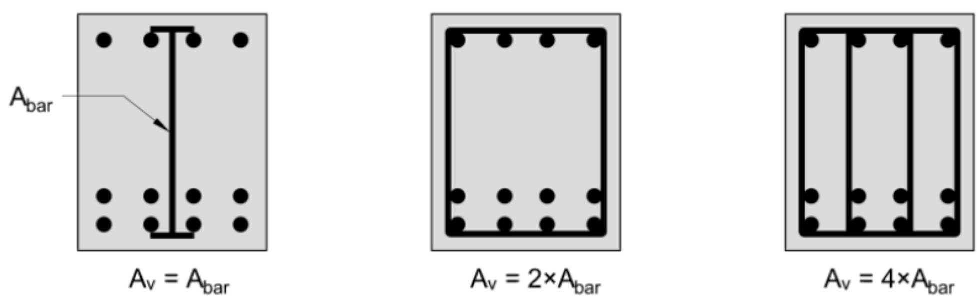


Fig 34.8 – Types of shear reinforcement and corresponding values of A_v

If this is not satisfied, we can calculate V_c using the equation with no shear reinforcements.

- If shear reinforcement is present, the maximum shear force carried in the truss is:

$$V_s = \frac{A_v f_y jd}{s} \cot(35°) \tag{28}$$

where s is the spacing of the shear reinforcement.

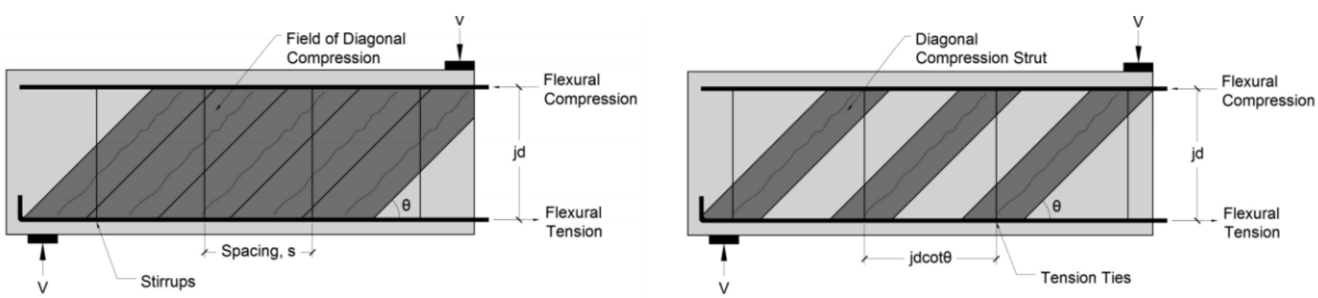


Fig. 34.9 – Diagonal stress fields in a cracked reinforced concrete beam (left) and Simplified truss model for concrete members subjected to shear (right)

- The **shear strength** V_r of the member is given by:

$$V_r = V_c + V_s \tag{29}$$

- The limit at which concrete crushes is:

$$V_{\text{max}} = 0.25 f_c' b_w jd \tag{30}$$

- The concrete will fail when:

$$V \geq \min\{V_r, V_{\text{max}}\} \tag{31}$$

- For design, we want to pick V_r to satisfy:

$$V_r = 0.5V_c + 0.6V_s \leq 0.5V_{\text{max}} \tag{32}$$

  where the constants represent the factors of safety. If a given design is not safe, here are the things that can be tried:

  – If $V \geq 0.5V_{\text{max}}$, the cross section needs to be resized.

  – If $V \geq 0.5V_c$, then reinforcements need to be made.

  – If $V \geq 0.5V_c + 0.6V_s$, then the spacing needs to be changed to:

$$s = \frac{0.6 A_v f_y jd \cot(\theta)}{V - 0.5 \times 0.18\sqrt{f_c'} b_w jd} \tag{33}$$

> **Idea**: In real life, cracks happen not at the highest shear, but instead a distance d away from it. Therefore, the maximum shear force we are designing for is given by:
>
> $$V_{\text{design}} = V_{\text{support}} - wd \tag{34}$$
>
> where w is the weight distribution. If $V_{\text{design}} \leq V_c$, no shear reinforcement is needed.

3.4 Prestressed Kuancrete

- The stress σ_c in prestressed concrete with concentric tendons is given by:

$$\sigma_{c,top} = -\frac{P}{A} - \frac{My_{\text{top}}}{I} \tag{35}$$

$$\sigma_{c,bot} = -\frac{P}{A} + \frac{My_{\text{top}}}{I} \tag{36}$$

- If the tendon was eccentric (offset from the center by a distance e), the stresses in the concrete are then:

$$\sigma_{c,top} = -\frac{P}{A} + \frac{Pey_{\text{top}}}{I} - \frac{My_{\text{top}}}{I} \tag{37}$$

$$\sigma_{c,bot} = -\frac{P}{A} - \frac{Pey_{\text{top}}}{I} + \frac{My_{\text{top}}}{I} \tag{38}$$