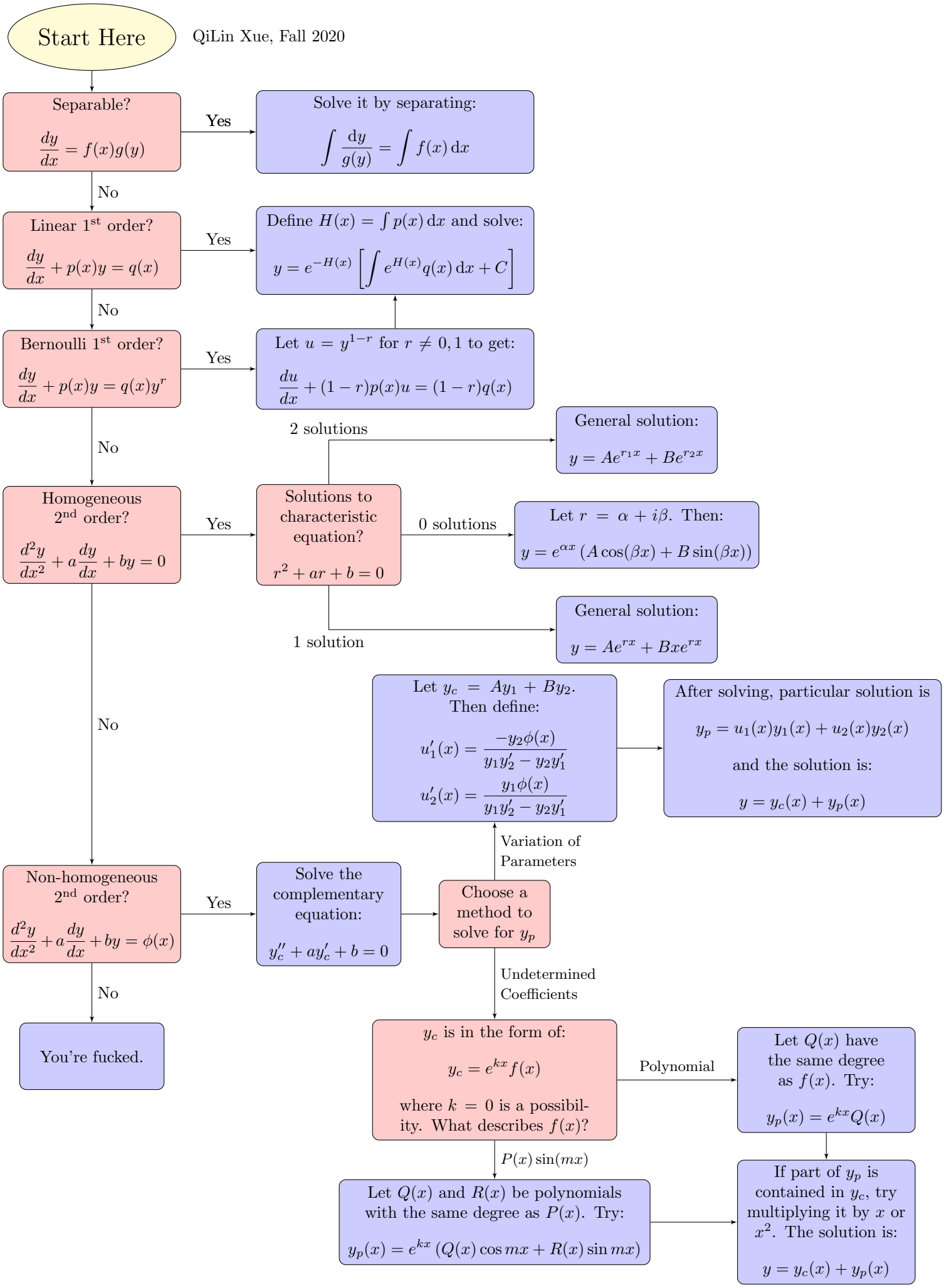
Start Here
QiLin Xue, Fall 2020
Separable?
$\frac{dy}{dx} = f(x)g(y)$
Yes
Solve it by separating:
$\int \frac{\mathrm{d}y}{g(y)} = \int f(x)\,\mathrm{d}x$
No
Linear 1st order?
$\frac{dy}{dx} + p(x)y = q(x)$
Yes
Define $H(x) = \int p(x)\,\mathrm{d}x$ and solve:
$y = e^{-H(x)}\left[\int e^{H(x)}q(x)\,\mathrm{d}x + C\right]$
No
Bernoulli 1st order?
$\frac{dy}{dx} + p(x)y = q(x)y^r$
Yes
Let $u = y^{1-r}$ for $r \neq 0, 1$ to get:
$\frac{du}{dx} + (1-r)p(x)u = (1-r)q(x)$
No
Homogeneous 2nd order?
$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$
Yes
Solutions to characteristic equation?
$r^2 + ar + b = 0$
2 solutions
General solution:
$y = Ae^{r_1 x} + Be^{r_2 x}$
0 solutions
Let $r = \alpha + i\beta$. Then:
$y = e^{\alpha x}\left(A\cos(\beta x) + B\sin(\beta x)\right)$
1 solution
General solution:
$y = Ae^{rx} + Bxe^{rx}$
No
Non-homogeneous 2nd order?
$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = \phi(x)$
Yes
Solve the complementary equation:
$y_c'' + ay_c' + b = 0$
Choose a method to solve for y_p
Variation of Parameters
Let $y_c = Ay_1 + By_2$. Then define:
$u_1'(x) = \frac{-y_2\phi(x)}{y_1y_2' - y_2y_1'}$
$u_2'(x) = \frac{y_1\phi(x)}{y_1y_2' - y_2y_1'}$
After solving, particular solution is
$y_p = u_1(x)y_1(x) + u_2(x)y_2(x)$
and the solution is:
$y = y_c(x) + y_p(x)$
Undetermined Coefficients
y_c is in the form of:
$y_c = e^{kx}f(x)$
where $k = 0$ is a possibility. What describes $f(x)$?
Polynomial
Let $Q(x)$ have the same degree as $f(x)$. Try:
$y_p(x) = e^{kx}Q(x)$
$P(x)\sin(mx)$
Let $Q(x)$ and $R(x)$ be polynomials with the same degree as $P(x)$. Try:
$y_p(x) = e^{kx}\left(Q(x)\cos mx + R(x)\sin mx\right)$
If part of y_p is contained in y_c, try multiplying it by x or x^2. The solution is:
$y = y_c(x) + y_p(x)$
No
You're fucked.