MAT185 Tutorial 3

QiLin Xue

February 2, 2021

Note: The treatment of these tutorial questions are not always very rigorous. The general ideas however for a completely rigorous proof are provided and should not be difficult to complete.

1 Tutorial Problems

Problem One

(a) We want to be able to have a, b, c, and d such that:

$$5x^4 + 5x^3 + 5x^2 + 13x + 4 = ax^2 + ax + a + bx^3 + bx^2 + 2bx + cx^3 + cx^2 + c + dx^4 + 8dx \tag{1}$$

or more suggestively:

$$5x^4 = dx^4 \tag{2}$$
$$5x^3 = (b+c)x^3 \tag{3}$$
$$5x^2 = (a+b+c)x^2 \tag{4}$$
$$13x = (a+2b+8d)x \tag{5}$$
$$4 = a + c \tag{6}$$

Solving, we get $d = 5$, $a = 0$, $c = 4$, $b = 1$ using every equation except the fourth one. However, we quickly realize that $13 \neq 0 + 2(1) + 8(5)$ and as a result, it is not in the span.

(b) No, because quartic equations have five degrees of freedom but the subset S only has four degrees of freedom.

Problem Two

(i) False. Let $\boldsymbol{x}_1 = \hat{\boldsymbol{\imath}}$ and $\boldsymbol{y}_1 = \hat{\boldsymbol{\jmath}}$ in $\mathbb{R}^2$. Adding the two spans of these two vectors spans the entire vector space. However, the span of $\boldsymbol{x}_1 + \boldsymbol{x}_2$ only gives a straight line

(ii) True. The span of $U \cup W$ consists of linear combinations of vectors that are in $U \cup W$ which we have proved last time is a subspace and we have also proved that $U \cup W = U + W$.

Problem Three

Notice that since S is a subset, then we may have $\operatorname{span} S \notin S$ since S is not necessarily a subspace. However, $\operatorname{span} S \in V$ is a subspace. Suppose there is another subspace $W \in V$ such that $S \in W$. Then by definition of the subspace, linaer combinations of vectors in S, we must have $\operatorname{span} S \in W$. Let us then define another subspace $X = \operatorname{span} S$.

To summarize, we have shown that all subspaces of V contain $\operatorname{span} S$ and there is one subspace of V that is $\operatorname{span} S$. Therefore, the intersection of all subspaces must be $\operatorname{span} S$.

2 Tutorial Worksheet

Task 2.1

Yes it is possible:

$$\frac{1}{14}\boldsymbol{v}_1 + \frac{1}{14}\boldsymbol{v}_2 = \nVdash \tag{7}$$

where $\boldsymbol{v}_1, \boldsymbol{v}_2 \in S$.

Task 2.2

Yes it is possible:

$$-\frac{1}{12}\boldsymbol{v}_1 + \frac{1}{30}\boldsymbol{v}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{8}$$

Task 2.3

Yes, as per below reasoning.

Task 2.4

Let

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \boldsymbol{x}_1$$

and

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \boldsymbol{x}_2.$$

Then:

$$\begin{aligned} a\boldsymbol{x}_1 + b\boldsymbol{x}_2 &= a\left(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2\right) + b\left(b_1\boldsymbol{v}_1 + b_2\boldsymbol{v}_2\right) && (9) \\ &= (aa_1 + bb_1)\boldsymbol{v}_1 + (aa_2 + bb_2)\boldsymbol{v}_2 && (10) \end{aligned}$$

where a_1, a_2, b_1, b_2 are coefficients determined above. These are orthogonal to each other, so they span $\mathbb{M}_2$.