MAT292
Tutorial 1 Solution

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1. (a) $\dfrac{dI}{dx} = -AI$ and $I(0) = I_0$.

   (b) Linear, Separable, Autonomous

   (c) $I(x) = I_0 e^{-Ax}$

   (d) Either 1/cm or 1/m. Looking at dimensions, note that $\dfrac{dI}{I} = -A\,\mathrm{d}x$. Since the left hand side is dimensionless, the right hand side is dimensionless, so A has the inverse units of x.

2. (a) At $x = 1/A$, we have $I(x) = I_0/e$.

   (b) We have

$$I_1 = I_0 e^{-Ax_1} \tag{1}$$
$$I_2 = I_0 e^{-Ax_2} \tag{2}$$

   Taking the log, we have

$$\ln I_1 = \ln I_0 - Ax_1 \tag{3}$$
$$\ln I_2 = \ln I_0 - Ax_2. \tag{4}$$

   Their difference gives

$$\ln I_1 - \ln I_2 = Ax_2 - Ax_1$$

   so

$$A = \frac{1}{x_2 - x_1} \ln\left(\frac{I_1}{I_2}\right) \tag{5}$$

   (c) We have

$$I = I_0 e^{-A_1x_1 - A_2x_2} \tag{6}$$

3. (a) We have

$$\ln(I/I_0) = -A_1x_1 - A_2x_2. \tag{7}$$

   Solving for x_1 gives

$$A_1x_1 = \ln(I_0/I) - A_2x_2 \tag{8}$$

   and similarly:

$$A_2x_2 = \ln(I_0/I) - A_1x_1 \tag{9}$$

   (b) We measure the intensity at various points. The peak (or dip) will be where the center is, and where it levels off is where the sphere ends. The difference between these two points is the radius.

   (c) Looking at the radiation intensity through the healthy cells allows us to measure A_{healthy}.

   Then shining the radiation through the center of the sphere, we can use part (a) to find A_{cancer}, since we know $x_{\text{cancer}} = 2r_{\text{cancer}}$ from part (b).