MAT292
Tutorial 3 Solution

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1. (a) We have

$$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y \tag{1}$$

   (b) Equilibrium occurs at $y = K$.

   (c) We have

$$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y - H(y,t) \tag{2}$$

2. (a) The new ODE is

$$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y - Ey \tag{3}$$

   (b) The dimensions of H is fish/time and the units of E is $[T]^{-1}$.

   (c) Equilibrium occurs when $\frac{dy}{dt} = 0$, which occurs at

$$r - \frac{ry}{K} - E = 0 \implies y = \frac{r-E}{r/K} \tag{4}$$

   and $y = 0$.

   (d) We can rewrite the derivative as $\frac{dy}{dt} = r - (r/K + E)y$, such that the phase plot is a line. If y decreases, then dy/dt is positive which causes y to increase. If y increases, then dy/dt is negative which causes y to decrease. Therefore, it is stable.

   On the other hand, $y = 0$ is unstable.

   (e) When the fish population stays constant, we want $y' = 0$, so rearranging, we have

$$E = r - \frac{ry}{K} \tag{5}$$

   (f) We have $H(y,t) = Ey = E\left(\frac{r-E}{r/K}\right)$

   (g) We have $E = \frac{r}{2}$. This is a quadratic.

3. (a) We have

$$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y - h \tag{6}$$

   (b) We use the quadratic equation

$$y = \frac{-r \pm \sqrt{r^2 - 4\left(-\frac{r}{K}\right)(-h)}}{-2r/K} = K \cdot \frac{r \pm \sqrt{r^2 - 4rh/K}}{2r} \tag{7}$$

   since $h < rK/4$, real solutions exist.

(c) The quadratic is concave down so the first equilibrium is unstable and the second equilibrium is stable.

(d) Same thing as above.

4. (a) If $h > rK/4$, then there are no equilibrium. The fish population will monotonically decrease.

(b) At $h = rK/4$, there is only one rate of fishing such that the fish population will remain constant. There is no way for it to increase.

(c) If we let $H(E) = \frac{rK}{4}$, then

$$E\left(\frac{r-E}{r/K}\right) = \frac{4K}{4}. \tag{8}$$

We can solve for E in terms of r and K to figure out the effort that represents the transition from sustainable to unsustainable fishing.

(d) Trivially obvious.