MAT292 Tutorial 4 Solution

QiLin Xue

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- 1. (a) b, c should have units of $\frac{1}{\text{year}}$
 - (b) I expect it will diverge, unless both budgets are initially zero.
 - (c) We have

$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} 0 & b\\c & 0 \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix}$$
(1)

We verify that the RHS has units of $\frac{\text{dollars}}{\text{year}}$.

- (d) The equilibrium is (0,0) and is unstable.
- (e) We can compute the eigenvalues

 $\lambda^2 = bc \tag{2}$

so we have $\lambda = \pm \sqrt{bc}$.

(f) We substitute this in to get

$$\begin{bmatrix} by\\ cx \end{bmatrix} = \begin{bmatrix} \sqrt{bcx}\\ \sqrt{bcy} \end{bmatrix}$$
(3)

plugging in x = 1 gives the eigenvector

$$\begin{bmatrix} 1\\ \sqrt{c/b} \end{bmatrix} \tag{4}$$

and the second eigenvector is $\begin{bmatrix} 1 \\ -\sqrt{c/b} \end{bmatrix}$.

(g) We have

$$\begin{bmatrix} x \\ y \end{bmatrix} = Ae^{\sqrt{bct}} \begin{bmatrix} 1 \\ \sqrt{c/b} \end{bmatrix} + Be^{-\sqrt{bct}} \begin{bmatrix} 1 \\ -\sqrt{c/b} \end{bmatrix}$$
(5)

2. (a) We have $\sqrt{c/b} = 3$. The lines are



(b) The phase portrait looks like

Phase Portrait

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(c) No.

- (d) They will diverge.
- 3. (a) I understand.
 - (b) We have

$$\begin{bmatrix} x'\\y' \end{bmatrix} = \begin{bmatrix} -2 & 1\\1 & -2 \end{bmatrix} \begin{bmatrix} x\\y \end{bmatrix} + \begin{bmatrix} C\\C \end{bmatrix}$$
(6)

(c) Did via Wolfram Alpha. Equilibrium occurs at $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} C \\ C \end{bmatrix}.$ We then have

$$\begin{bmatrix} u'\\v'\end{bmatrix} = \begin{bmatrix} -2 & 1\\1 & -2 \end{bmatrix} \begin{bmatrix} u\\v \end{bmatrix}$$
(7)

and get

$$\begin{bmatrix} x \\ y \end{bmatrix} = Ae^{-3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + Be^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
(8)

(d) The solution to the nonhomogenous equation is

$$\begin{bmatrix} x \\ t \end{bmatrix} = Ae^{-3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + Be^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} C \\ C \end{bmatrix}$$
(9)

(e) False according to Parveer.