MAT292
Tutorial 4 Solution

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1. (a) b, c should have units of $\frac{1}{\text{year}}$

   (b) I expect it will diverge, unless both budgets are initially zero.

   (c) We have

   $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{1}$$

   We verify that the RHS has units of $\frac{\text{dollars}}{\text{year}}$.

   (d) The equilibrium is $(0, 0)$ and is unstable.

   (e) We can compute the eigenvalues

   $$\lambda^2 = bc \tag{2}$$

   so we have $\lambda = \pm\sqrt{bc}$.

   (f) We substitute this in to get

   $$\begin{bmatrix} by \\ cx \end{bmatrix} = \begin{bmatrix} \sqrt{bc}x \\ \sqrt{bc}y \end{bmatrix} \tag{3}$$

   plugging in $x = 1$ gives the eigenvector

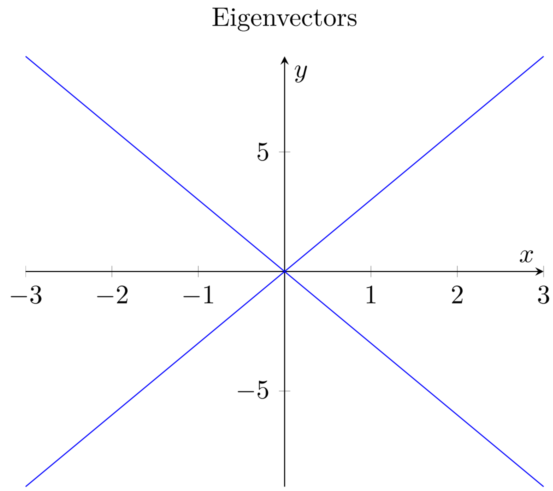
   $$\begin{bmatrix} 1 \\ \sqrt{c/b} \end{bmatrix} \tag{4}$$

   and the second eigenvector is $\begin{bmatrix} 1 \\ -\sqrt{c/b} \end{bmatrix}$.

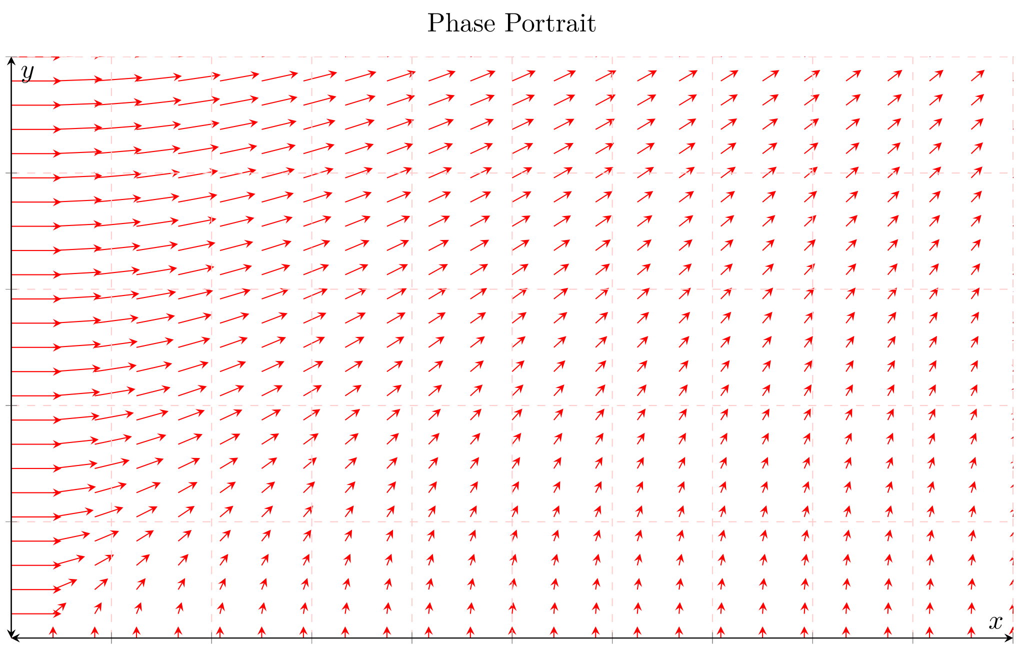
   (g) We have

   $$\begin{bmatrix} x \\ y \end{bmatrix} = Ae^{\sqrt{bc}t} \begin{bmatrix} 1 \\ \sqrt{c/b} \end{bmatrix} + Be^{-\sqrt{bc}t} \begin{bmatrix} 1 \\ -\sqrt{c/b} \end{bmatrix} \tag{5}$$

2. (a) We have $\sqrt{c/b} = 3$. The lines are



(b) The phase portrait looks like



(c) No.

(d) They will diverge.

3. (a) I understand.

(b) We have

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} C \\ C \end{bmatrix} \tag{6}$$

(c) Did via Wolfram Alpha. Equilibrium occurs at $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} C \\ C \end{bmatrix}$. We then have

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{7}$$

and get

$$\begin{bmatrix} x \\ y \end{bmatrix} = Ae^{-3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + Be^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{8}$$

(d) The solution to the nonhomogenous equation is

$$\begin{bmatrix} x \\ t \end{bmatrix} = Ae^{-3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + Be^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} C \\ C \end{bmatrix} \tag{9}$$

(e) False according to Parveer.