PHY294: Practice Problems
Problem Set 2 Solutions

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(9.1) The magnitude is $S = \sqrt{s(s+1)}\hbar = \sqrt{3}/2\hbar$ where $s = 1/2$ for an electron, and $S_z = \pm\frac{1}{2}\hbar$. Therefore, the angle between $\vec{S}$ and e_z is

$$\theta = \arccos\frac{S_z}{S} = 54.736^\circ. \tag{1}$$

(9.3) (a) We have $S_z = m_s\hbar h$ where $m_s \in \{1, 0, -1\}$. Therefore, it can take on three possible values.

(b) Angle can be calculated using same method as previous question.

(c) The minimum possible angle is when S_z is closest to S, i.e. when $m_s = 1$. This gives

$$\theta_{\text{min}} = 45^\circ. \tag{2}$$

(9.5) This corresponds to the $1s^2 2s^2 2p^6$ state.

- For $(n, \ell) = (1, 0)$, we have $m = 0$, so there are two possible values for m_s.
- For $(n, \ell) = (2, 0)$, we have $m = 0$, so there are two possible values for m_s.
- For $(n, \ell) = (2, 1)$, we have $m \in \{-1, 0, 1\}$, so there are six possible values for m_s.

(9.8) The moment of inertia of a ball is $I \approx mr^2$ (Note that the distribution of mass may change this, possibly by a factor of $\frac{2}{5}$, but it's the order of magnitude that counts). The angular momentum is then:

$$mr^2\frac{v}{r} \sim \hbar \implies \frac{v}{c} \sim \frac{\hbar}{mrc} \approx 40,000, \tag{3}$$

which clearly isn't possible!

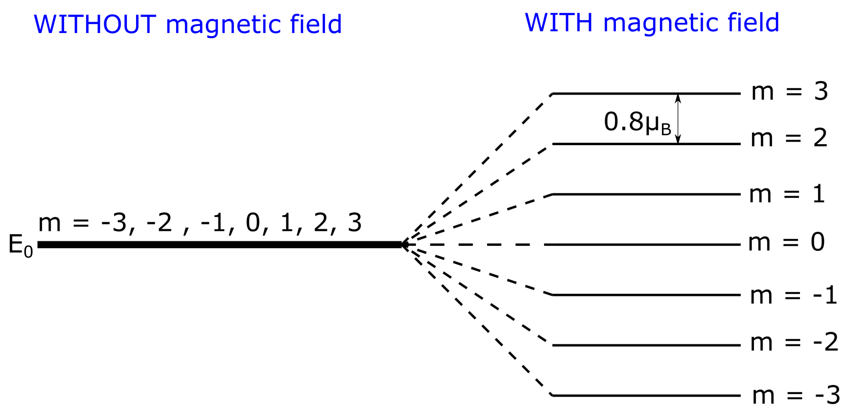
(9.9) (a) The magnetic moment is $\mu = I(\pi r^2) = 1.26 \times 10^{-4}\,\text{A m}^2$

(b) The torque is $\tau = \mu B \sin\theta = 1.89 \times 10^{-4}\,\text{A m}^2\,\text{T}$.

(c) The potential energy is $U = -\mu B\cos\theta$. If B is flipped, then $\cos\theta$ goes from 1 to -1, and therefore $\Delta U = -2\mu B = -3.78 \times 10^{-4}\,\text{J}$.

(9.11) Similar to above, we have $\mu = I(\pi r^2) \implies I = \frac{\pi r^2}{\mu} = 3.18 \times 10^{-4}\,\text{A}$.

(9.15) (a) See below



(b) The energy difference between adjacent levels is $\mu_B B$, where $\mu_B = 9.27 \times 10^{-24}\,\mathrm{J\,T^{-1}}$ is the Bohr magneton.

(9.17) (a) Very similar to the above diagram except in the higher level there are $2l+1=5$ sub-levels while in the lower level there are $2l+1=3$ sub-levels.

(b) Since $\Delta E \propto L_z$, which is dependent on m_i, m_f, let us look at the number of distinct differences $m_f - m_i$ (now this is just a statistics problem!) If $m_i = 1$, there are five possible differences. If $m_i = 0$, there are still five possible differences, but only one of them will be new. The same goes for $m_i = -1$. Therefore, there are $5+1+1=7$ possible photon energies.

(c) To see why this is true, note that

$$E_f = E_{0,f} + \mu_B B m_f$$
$$E_i = E_{0,i} + \mu_B B m_i$$

and so

$$E_\gamma = E_f - E_i = (E_{0,f} - E_{0,i}) + \mu_B B(m_f - m_i). \quad (4)$$

The photon energy is dependent on $m_f - m_i$. Since this can only take on three values, the photon energy can only take on three values.

(9.19) (a) For the anomalous Zeeman effect, we have

$$E_{\text{diff}} = 2\mu_B B = 1.3 \times 10^{-23}\,\mathrm{J} \quad (5)$$

(b) Using the relationship $E_\gamma \lambda = hc$, we determine the wavelength to be on the order of magnitude of 1 cm, which corresponds to microwaves.