Studying Fluid Flow Through Channels of Various Shapes

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I. INTRODUCTION

The purpose of the lab is to apply fluid mechanics to model fluid flow through channels of varying widths and shape. In particular, we look at the relationships between flow speed and pressure. In incompressible flow, along any streamline, the quantity

$$\frac{1}{2}\rho v^2 + \rho g z + P = \text{constant} \quad (1)$$

is conserved [1], where ρ is the density, v is the speed, g is the gravitational acceleration, z is the height (relative to an arbitrary point), and P is the static pressure at a given point. In a typical microfluidics experiment, there is a syringe filled with water connected to a thin tube that exits a distance h below the syringe and both the top of the syringe and the exit is exposed to atmospheric pressure. Suppose there is a streamline that starts at the surface and ends at the exit, and compare the above quantity at both the starting and ending points. Let v_1 be the speed at which the surface is lowering. If we assume v_1^2 is small, then we arrive at Toricelli's Law [1]:

v_{exit} = p2gh (2)

At steady flow, the amount of fluid that flows past a certain cross section should remain constant with respect to time. From conservation of mass, we can show that

Integral v_1 dA_1 =integral v_2 dA_2 (3)

Where A_i and v_i refer to the cross sectional area and speed respectively at some point i. If we assume that the flow is constant in any given cross section, this reduces to

$$A_1 v_1 = A_2 v_2. \quad (4)$$

However, Bernoulli's equation may not apply for viscous and turbulent flow. Theoretical and computational methods to model turbulent and viscous flow are often complex, so we are also interested in how much the ideal laminar flow approximation applies

II. METHOD

The procedure from the laboratory manual was followed [2], and data analysis was done using the software *ImageJ* [3]. Using the *Find Edges* feature, it allowed the streaks to be more defined, which was especially helpful when dealing with overlap, as shown in figure 1. We selected the beginning and ends of all the relevant streaks, and the program was able to output a table of x and y values for us to analyze.

The default unit *ImageJ* uses is a 300 pixel : 1 unit, and the uncertainty in position when following the above method to measure the streak is 0.1 unit = 30 pixels. We obtained this value by selecting the start and end point of a given streak numerous times and recording the standard deviation in length. This uncertainty is caused by blurring towards the end of each streak. This should be independent of how long the streak is, so unless otherwise stated, this is the measurement uncertainty we will be using.

The length was converted to the actual length by using the scale that appears in each question. The uncertainty of the length of the given scale is 0.02 units, calculated in the same way as above. The reason this is smaller is because the ends of the scale is more well defined.

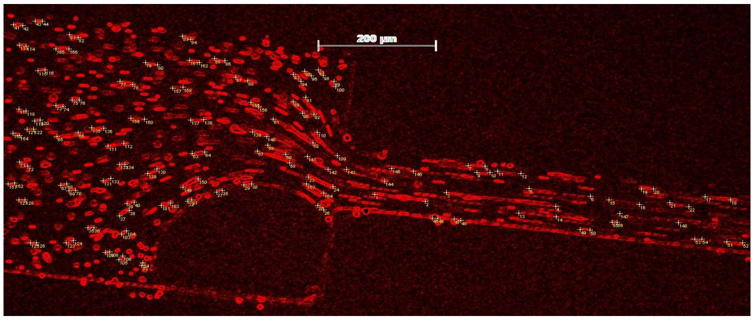

FIG. 1: An example of an ImageJ output. The red lines show the streaks and the yellow crosses show user markings. Note that the edges are outlined.

III. DISCUSSION

We analyzed the flow of fluid for five cases to observe three overall phenomena: flow in a straight channel, flow through a bend, and flow through varying cross sections. We will discuss how well our model holds in each of the three cases.

A. Flow Through A Straight Channel

In this scenario, fluid flow moves through a channel of diameter 130 ± 7μm. Figure 2 shows the relationship between the speed of the streak and its distance from one of the edges.

We chose a quadratic model with one parameter as we are using the Hagen-Poiseuille equation to model viscous flow, which tells us the speed distribution is given by[4]:

$$v(r) = -\alpha(R^2 - r^2) \qquad (5)$$

where α is some constant that depends on the geometry of the pipe and the properties of the fluid. Note that there is only one parameter α, so this prevents overfitting. Although the model is not perfect, with R^2 = 0.535, it does show the presence of a nonlinear effect and the fact that the maximum speed occurs near the middle, which is what we expected.

Visually, we observe the flow to be laminar as the paths that the beads take are smooth streamlines. We have also seen that the speed distribution roughly follows a quadratic, which is what the Hagen-Poiseuille relationship predicts.

We are also interested in how various factors such as the height of the syringe or shape of the tube effects the speed. However, this is a complex task as the speed greatly varies even within a single cross section! Therefore, for simplicity, we will only focus on the speed in the center, unless otherwise indicated so the speeds in table I are recorded at the center of the tube only. We see that as the height

Height (cm)	Speed (mm/s)
10 ± 3	3.4 ± 0.5
15 ± 3	4 ± 1
20 ± 3	4.9 ± 7

TABLE I: Average speed for the straight tube case when changing the height of the gravity head. Measurements were averaged and the standard deviation was taken to be the uncertainty. increases, the speed also increases. Since we only have three data

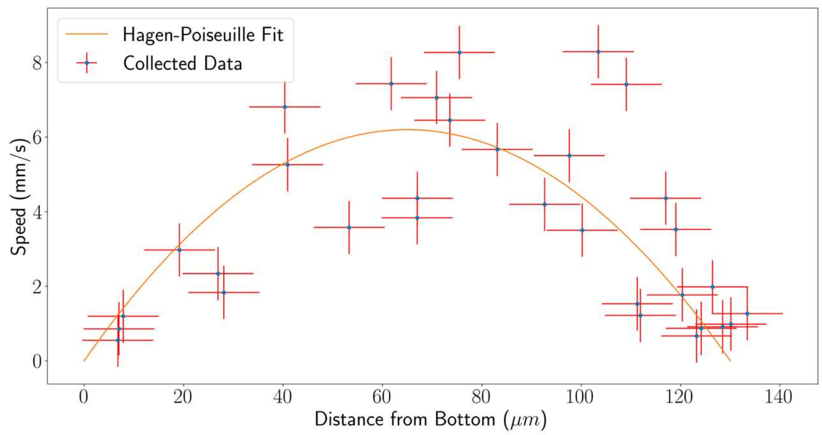

FIG. 2: The plot of the collected data along with a line of best fit. The speed clearly drops off towards the edges but near the middle there is a range of speed distributions.

points, we cannot draw a conclusion on how the height of the gravity head affects the speed of the fluid. This is because an increase in height applies a greater pressure. Put it more strictly, consider two points on a streamline: one from the surface of the water at the syringe (point 1) and the other at the point of interest in the straight channel (point 2). If point 1 is a height z above the tube, then from Bernoulli's Principle:

$$P_{\text{atm}} + \rho g z = P_2 + \frac{1}{2}v^2 \tag{6}$$

where we have assumed that since the area of the tube is much greater than the area of the tube, the speed at which the water level drops is negligible. We can expect that an increase in the height z will cause an increase in the speed v.

Also note that changing the volume has the same effect as changing the height. For example, if a volume of ΔV is added to a syringe of cross sectional area A, it is essentially equivalent to raising the surface of the water by a height $(\Delta V)/A$.

B. Flow Through a Bend

We also considered two bends: one is sharp and forms an "L" shape while the other is smooth and forms an "S" shape. The cross sectional area of both these tubes remain constant, so we should expect the speeds to remain constant. Figure 3 shows how the streak lines look in these two cases. Table II and Notice that in both the smooth and

Location	Speed in S curve (mm/s)	Speed in L curve (mm/s)
Start	1.1 ± 0.2	1.3 ± 0.1
Middle	1.0 ± 0.1	1.3 ± 0.1
End	1.1 ± 0.1	1.2 ± 0.1

TABLE II: Average speed for the straight tube case when changing the height of the gravity head. Measurements were averaged and the standard deviation was taken to be the uncertainty.

sharp bends, the speeds before, during, and after the bend all agree with each other. This is both expected, since for constant areas, the continuity equation becomes $v_1 = v_2$, yet it is still interesting, since we looked at the speed near the center of the pipe only, and ignored the flow near the edges. This further supports the idea that flow is laminar. This is because in laminar flow, given constant cross section, all particles should travel parallel to the axis of the pipe.

Another interesting phenomena is the velocity profile during the bend. The location at which maximal speed occurs is no longer in the center, but skewed towards the outside edge.

First, we can expect the fluid to go around the curve in roughly the same time. Since the fluid that travels near the outside edge needs to travel a further distance, it travels at a faster speed. Note that this isn't exactly right, as this is only true if the flow is classified as *potential flow*, i.e. $\nabla \times v = 0$, which isn't necssarily true, even for laminar flow. However, we should still expect there to be some speed increase.

We can further back this up with our lived experience in everyday lives. In curved rivers, there is often more erosion occurring on the

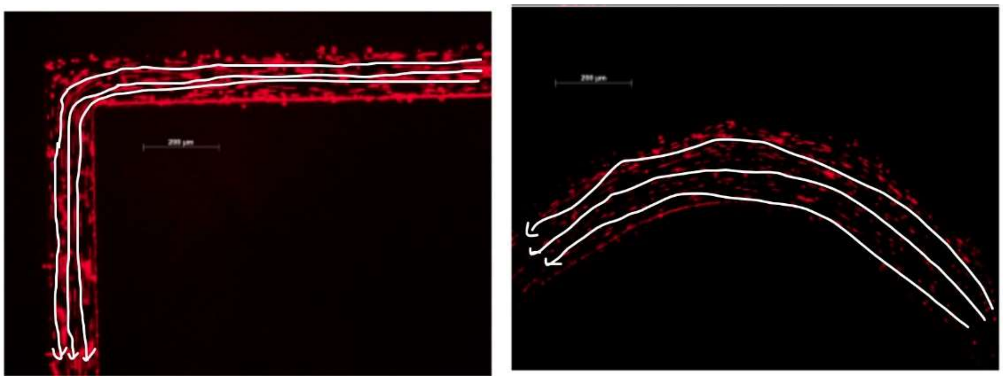

FIG. 3: Photos of both the L shaped bend (above) and the S shaped bend (below) while the fluid is moving through. The flow is moving from right to left. Paths are drawn in white.

outside edge of a bend since the water is moving faster. As a result, the fact that maximum speed occurs near the edge in our microfluidics experiment should not be a surprise.

However, since the fluid still has some nonzero viscosity, it is important to note that the maximum speed does not occur exactly at the edge. There are two competing effects: as we move closer to the outside edge, the faster the fluid moves due to the roughly equal transit time approach, but it also experiences stronger viscous forces.

The point at which these two competing interactions balances out is where the maximum speed occurs, and interestingly, this location is different for the L and S shaped curves. For the S shaped curve, this point occurs extremely close the the edge while for the L shaped curve, this point occurs relatively far from the edge. In fact, there is a portion of the fluid near the corner of the L shaped curve that appears almost stationary. We suspect that there are turbulent effects here, which is likely caused by water that comes from the top channel hitting the leftmost wall, which exerts a force on the fluid element. This force has no component perpendicular to the flow, so the fluid gets pushed back to the right, and creates a source for turbulence. To be clear, the motion of the beads are drawn in figure 3.

C. Effects of Changing Channel Width

When we widen a channel, mass conservation Av = constant should still hold. In this section, we look at two methods of widening the channel similar to how we examined bends: rapidly and gently. These two channels are shown in figure 4 The relationship between area and flow speed in the gradual case is graphed in figure 5. It is expected that as the area increases, the speed decreases. However, it does not appear that the relationship $v = \frac{\alpha}{A}$ holds too well, where A is the cross sectional area and α is some constant. The reason is likely because we collected data only near the center.

In the previous section, we saw that $A_1v_1 = A_2v_2$ held if we picked speed in the center since the cross sections remained constant. However, as the area increases, streamlines are no longer parallel to each other and this graph shows that the same assumption may no longer hold.

The case where the opening suddenly opens is interesting since it is broken up into three regions. The first region is the incoming fluid from the right in a constant width tube. The second region is in the middle where near the bottom, we have a gradual transition (and thus we expect behaviour similar to the gradual case above). Near the top, we have a sudden transition. In this region, and especially near the corner, we can see evidence of some turbulent flow. However in the gradual transition, we did not see any clear indicators of turbulent flow: only laminar viscous flow.

We can compare the speed of the fluid in both the inlet and outlet channels in table III. Note that our expected values were very generally inconsistent with the expected values. Although the uncertainties do overlap, the model is not as accurate because we are only looking at one portion of the tube, which is the center. A more formal approach would be to compute

$$\int v\, dA \qquad (7)$$

perhaps by using a symmetry approach. We did not find any significant discussion to be made when changing the level of the grav-

Location	Speed - Gradual (mm/s)	Speed - Abrupt (mm/s)
Inlet	2.2 ± 0.5 ± 0.5	4 ± 1
Outlet	0.5 ± 0.2	1.1 ± 0.3
Outlet (Expected)	0.2 ± 0.3	0.5 ± 0.3

TABLE III: Summary of measured and expected speeds of both the gradual and abrupt changes in diameter.

ity head. Increasing the volume in the syringe is equivalent to increasing the height of the syringe, which has the same effect as in the previous section: the speed of the water increases. Again, since we were only able to perform the experiment on a small set of heights, any sort of modelling beyond this simple statement would lead to overfitting.

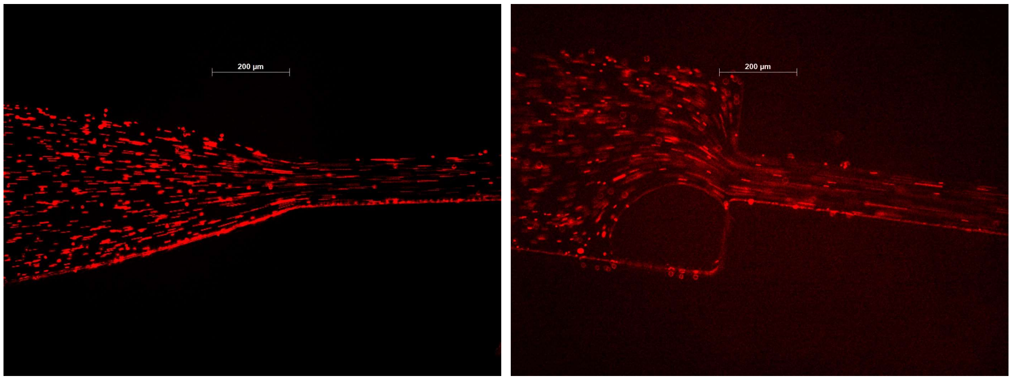

FIG. 4: Time exposure pictures of the streaks in the gradual case (above) and the sudden increase in area case (below).

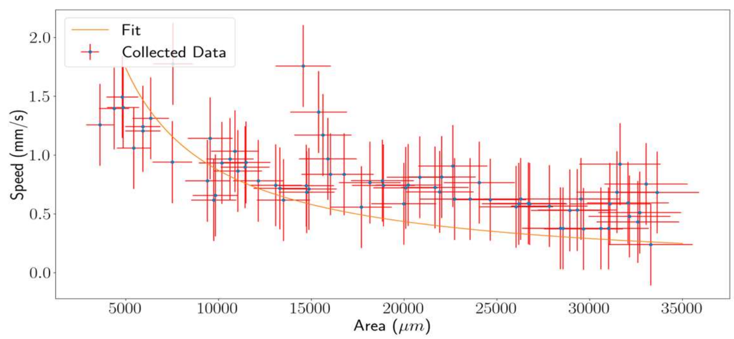

FIG. 5: A plot of the area versus the flow speed, where the data is recorded once the width changes. While an inverse relationship is seen, the fit Av = constant does not seem to hold particularly well.

IV. CONCLUSION

In conclusion, we have demonstrated introductory fluid mechanics concepts using microfluidics and found that in most cases, we observed laminar flow, and thus Bernoulli's principle applied. Turbulent flow was mostly observed when there was an abrupt transition such as a sharp corner. Mass conservation

$$A_1v_1 = A_2v_2 \tag{8}$$

was also demonstrated for some cases, but we found that it was difficult to demonstrate in the general case since the velocity profile is not uniform, especially when there is a change in the cross section.

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[2] Bryan Keith et al. Lab manual: Introduction to microfluidics.

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