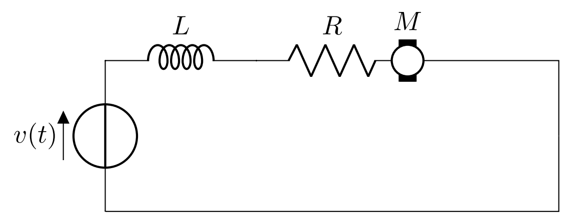
AER372: Control Systems
Prelab 4

Spring 2023

1. We have the following circuit:



From Kirchoff's Law, we have

$$v(t) - L\dot{i}(t) - i(t)R - K_e\dot{\theta}(s) = 0 \tag{0.1}$$

where $v_d = -K_e\dot{\theta}(s)$ is the voltage drop across the motor, given by Faraday's Law. We can then write the differential equation for the motor as

$$\tau_m = K_t i(t) = \left(\underbrace{J_r + \frac{1}{2}m_d r_d^2}_{J}\right)\ddot{\theta}(s) \tag{0.2}$$

where J is the moment of inertia of the motor and the load combined.

2. Taking the Laplace Transform, we get

$$V(s) = LsI(s) + I(s)R + sK_e\Theta(s) \tag{0.3}$$
$$K_tI(s) = s^2J\Theta(s). \tag{0.4}$$

Solving for $I(s)$ and substituting that into the first equation gives

$$V(s) = \frac{Ls^3J}{K_t}\Theta(s) + \frac{s^2JR}{K_t}\Theta(s) + sK_e\Theta(s) \tag{0.5}$$

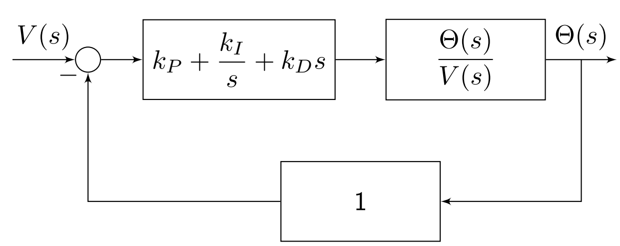
and solving for $\Theta(s)/V(s)$ gives

$$\frac{\Theta(s)}{V(s)} = \frac{1}{\frac{Ls^3J}{K_t} + \frac{JR}{K_t}s^2 + K_es}. \tag{0.6}$$

3. Dividing both the numerator and denominator by R, we can set $\dfrac{Ls^3J}{K_tR} \to 0$ to get the second order system

$$\frac{\Theta(s)}{V(s)} = \frac{K_t}{RJs^2 + K_eK_ts}. \tag{0.7}$$

4. We have



5. We're interested in the open-loop transfer function. If we multiply it out, we have

$$G(s)D(s) = \left(k_P + \frac{k_I}{s} + k_D s\right)\frac{K_t}{RJs^2 + K_e K_t s} = \frac{K_T(k_I + k_p s + k_D s^2)}{s^2(K_e K_T + JRs)} \tag{0.8}$$

is type 2, so it can track a step and ramp input with zero steady state error. To compute the SS error for a quadratic input, we can use the final value theorem,

$$K_a = \lim_{s\to 0} s^2 G(s)D(s) = \frac{K_T k_I}{K_e K_T} = \frac{k_I}{K_e} \tag{0.9}$$

so the steady state error is

$$e_{ss} = \frac{1}{K_a} = \frac{K_e}{k_I}. \tag{0.10}$$