AER372: Easy Solution to Pendulum on a Cart

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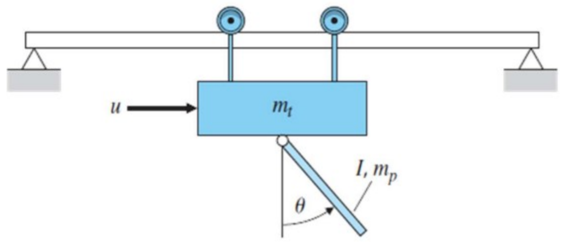
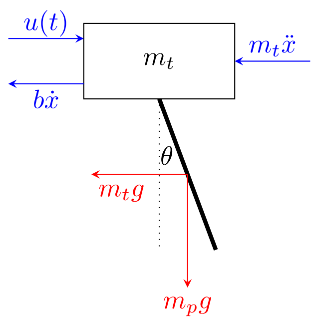
Problem

Consider a freely rotating rigid pendulum of mass m_p, length 2ℓ and moment of inertia I about its center of mass, attached to a cart of mass m_t. The cart is being pushed with a force $u(t)$ and the damping constant is b.



Simple Solution

We can describe the system by the coordinates (x, θ). We apply d'Alambert's Principle. Specifically, we change into an accelerating frame of reference by adding a force $-m_t\ddot{x}$ to all bodies, such that m_t is stationary. The free body diagram now looks like this, where we have neglected internal forces:



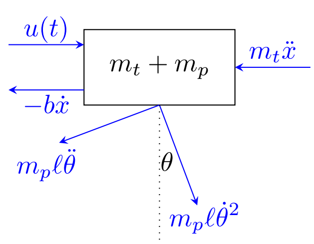
Because the pivot point is stationary and because angles do not change under a different frame of reference, we can balance torques about the pivot, i.e.

$$\boxed{(I + m_p\ell^2)\ddot{\theta} = -m_p g\ell\sin\theta - m_t g\ell\cos\theta}. \tag{1}$$

Next we consider forces on m_t. Treating the block as its own system, we now have forces the pendulum exerts on the block. We can do this a clever way by applying d'Alambert's Principle a second time. The pendulum's acceleration can be decomposed into two components:

- Component 1: radial acceleration towards the pivot point with magnitude $\ell\dot{\theta}^2$.
- Component 2: tangential acceleration, perpendicular to the pendulum, with magnitude $\ell\ddot{\theta}$

Then the free body diagram in this new accelerating reference frame is, (where we are able to combine m_t and m_p since they are both stationary)



This is static, so the net force is zero. Decomposing everything to be in the x-direction, we have:

$$\boxed{m_t\ddot{x} = u(t) - b\dot{x} + m_p\ell\ddot{\theta}\cos\theta - m_p\ell\dot{\theta}^2\sin\theta}. \tag{2}$$

d

(a) If we have repeated roots, then we can factor

$$H(s) = \frac{\omega_n^2}{(s + \omega_n)^2}, \tag{3}$$

i.e. $\zeta = 1$.